Table 1 Comparison of efficiencies: (a) Eq. (13), (b) Eq. (14) with three terms, (c) Eq. (14) with four terms

Method	Λ=5	Λ=10	Λ=20
a	0.7008	0.8287	0.9077
b	0.7013	0.8290	0.9078
c	0.7012	0.8289	0.9078
Baclic, 1985	0.7011	0.8288	0.9077

Baclic is very accurate, but it involves the calculation of the values of some Bessel functions, which makes it difficult to be used in practice. The simple form is also useful in obtaining simple analytical solutions for economic optimization of a regenerator, where an economic target function that depends both on the efficiency and heat transfer area are maximized or minimized. It should be noted that these simple formulas are restricted to small values of Π. The more complicated formula presented by Baclic is valid for all values of Π.

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On the Effective Driving Force for Transport in Cooling Towers

M. Sadasivam[1,2] and A. R. Balakrishnan[1,3]

Nomenclature

a = interfacial area, m^2/m^3
b_o = slope of saturated air enthalpy, J/(kg·°C)
B'' = defined in Eq. (4), J/(kg·°C)
C_p = specific heat, J/(kg·°C)
C_s = humid heat, J/(kg·°C)
G = superficial gas mass velocity, $kg/(m^2 \cdot s)$
h = convective heat transfer coefficient, $W/(m^2 \cdot °C)$
H = enthalpy, J/kg; H^*: apparent enthalpy, J/kg
k_y = gas phase mass transfer coefficient, $kg/(m^2 \cdot s)$
L = superficial liquid mass velocity, $kg/(m^2 \cdot s)$
Le = Lewis number
NTU = number of transfer units
q = heat flux, W/m^2
r = convective Lewis number
T = temperature, °C
Y = absolute humidity, kg H_2O/kg dry air
Z = height of tower, m
α = defined in Eq. (4), J/(kg·°C)
β = defined in Eq. (4), J/(kg·°C)
ϵ = effectiveness
λ = latent heat of vaporization, J/kg

Subscripts

A = component A, water vapor
B = component B, dry air
G = air
H = heat transfer
i = interface condition
L = (liquid) water
m = mass transfer
o = reference point
s = sensible heat transfer
w = wet bulb

Since both heat and mass transfer take place in cooling towers, Merkel (1926) suggested that the driving force for heat transfer in cooling towers is the enthalpy difference of the wet air at the interface and in the bulk. Cooling tower design equations have been obtained analytically based on Merkel's (1926) approach and by expressing the saturated air enthalpy as a linear function of temperature (ϵ–NTU method of Berman, 1961, and Jaber and Webb, 1989) or as a quadratic function of temperature (Foust et al., 1979; Fahim et al., 1985). In all these studies the Lewis relation has been assumed to be valid. In this study an apparent air enthalpy is defined, thereby obviating the need to invoke the Lewis relation, and the use of this apparent enthalpy in the ϵ–NTU method of cooling tower design is demonstrated.

The so-called Lewis relation can be distinguished from the Lewis number by considering the following parameter:

$$r = \frac{h_G a_H}{C_s k_y a_m} \tag{1}$$

r is related to the Lewis number, Le, defined as the ratio of the Schmidt number to the Prandtl number, by empirical correlations. For the air–water system, Lewis (after W. K. Lewis) showed that the ratio of the gas-phase heat transfer coefficient to the gas-phase mass transfer coefficient is approximately equal to the humid heat of the wet air. This implies that r should be unity for the air–water system, and is commonly called the "Lewis relation." However, this is only approximately true, for as Hensel and Treybal (1952) have pointed out, r can be as high as two at very low liquid flow rates. However, only when r is unity, is it called the Lewis relation. Some investigators have called r the convective Lewis number, Le_c, when r takes on values other than unity. r is also sometimes referred to as the psychrometric ratio (when $a_H = a_m$, which is quite common). Different authors have used different definitions for the Lewis number, Le. Webb (1990) compared all these definitions and concluded that the

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ratio of thermal diffusivity to mass diffusivity is the most meaningful one. Webb (1988) has described the magnitude of the possible errors associated with cooling tower design procedures. It may be concluded from the above that the introduction of the Lewis relation into the balance equations results in error. This is because even for air–water systems, the convective Lewis number varies widely.

Analysis

A mass and energy balance over a differential segment in the cooling tower gives

$$G_B dH_G = k_y a_m (H^*_{Gi} - H^*_G) dZ = dq \qquad (2)$$

where H^*_G, the apparent enthalpy, is given by

$$H^*_G = C_s r T_G + \lambda_{A0} Y_A \qquad (3)$$

Assuming the Lewis relation is valid for the air–water system, i.e., $r = 1.0$, results in the apparent enthalpy collapsing to the absolute enthalpy, which is the Merkel approach, i.e., the driving potential for heat transfer in cooling towers is the enthalpy difference of the wet air at the interface and at the bulk. This is true if, and only if, r is unity, i.e., the Lewis relation is satisfied. On the other hand Eq. (2) is based on the effective or apparent enthalpy driving potential, H^*_G. This effective or apparent enthalpy differs from the actual enthalpy in that it has included in it the

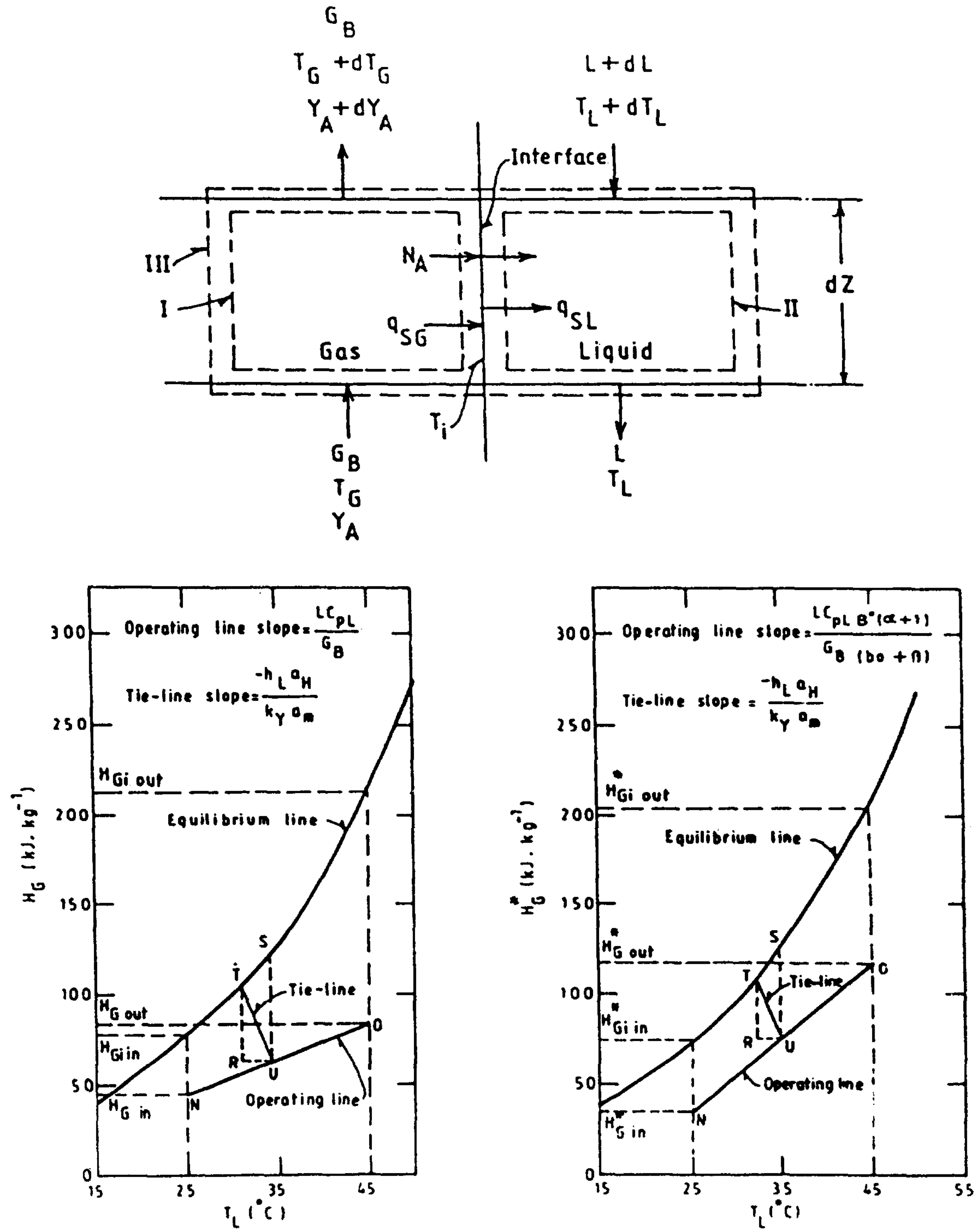

Fig. 1 Operating diagram of the cooling tower: (*a*) on the air enthalpy versus liquid temperature diagram; (*b*) on the air apparent enthalpy versus liquid temperature diagram; (*c*) schematic of a differential segment of a cooling tower

convective Lewis number, r. From Eq. (3) it is seen that the apparent enthalpy is less than the actual enthalpy when r is less than one and vice versa.

Figure 1 shows the operating diagram of cooling towers in terms of the actual enthalpy and the apparent enthalpy. It is seen that when the apparent enthalpy is used the equilibrium line gets shifted closer to the operating line, i.e., driving potential is reduced. All the previous studies on cooling towers have indeed shown that using absolute enthalpies results in underestimation of the cooling tower height. The cooling tower design problem can best be handled on an operating diagram drawn on the apparent enthalpy (H_G^*) versus temperature chart (Fig. 1). This requires a relationship between the apparent and actual enthalpies of the air. This can be obtained using the tie-line relationship (see Fig. 1). Point U on the operating line corresponds to any position in the column, and point T (on the equilibrium line) represents the interface conditions. The distance TR is the enthalpy driving force ($H_{Gi} - H_G$) within the gas phase. Point S corresponds to a position where the heat transfer resistance is neglected on the liquid side, i.e., $T_i = T_L$ (or the slope of $h_L a_H / k_y a_m$ is infinite). However, as Marseille et al. (1991) have shown, the liquid side heat transfer cannot be neglected in cooling tower design without incurring error. The slope of the operating line is $L.C_{pL}/G_B$ (from the overall enthalpy balance). The slope of the tie-line is obtained from the liquid side enthalpy balance. If the operating diagram is constructed in terms of the saturated apparent enthalpy (H_{Gi}^*) versus temperature, Fig. 1(b), the slope of the tie-line will remain the same as for H_{Gi} versus T_L. Using the liquid side and overall enthalpy balance and representing the saturated enthalpy H_{Gi} and the saturated apparent enthalpy H_{Gi}^*, as a linear function of T_i, it can be shown

$$dH_G = \frac{(b_0 + \beta)}{(\alpha + 1)B''} dH_G^* \tag{4}$$

where $\beta = h_L a_H / k_y a_m$; $\alpha = \beta . G_B / L . C_{pL}$; $B'' = f' - \{\alpha/(\alpha + 1)\} \cdot (b_0 + \beta) + \beta$; and b_0 and f' are the slopes of the H_{Gi} versus T and H_{Gi}^* versus T curves, respectively. Introducing Eq. (4) in the overall enthalpy balance gives

$$dq = G_B \frac{(b_0 + \beta)}{(\alpha + 1)B''} dH_G^* = LC_{pL} dT_L \tag{5}$$

Following an approach similar to the ϵ–NTU method of Jaber and Webb (1989), it can be shown that $L \cdot C_{pL}/f'$ is the water capacity rate and $G_B(b_0 + \beta)/(\alpha + 1)B''$ is the air capacity rate. These water and air capacity rates differ from the definitions of Jaber and Webb (1989), in that while f' in the present study is the slope of the apparent saturated air enthalpy versus temperature curve (H_{Gi}^* versus T), Jaber and Webb (1989) used the true saturated air enthalpy versus temperature curve (H_{Gi} versus T). Furthermore, in the present study, the capacity rate of the fluid is a function of the convective Lewis number. However, if the Lewis relation is invoked, i.e., $r = 1$, the present definitions of water and air capacity rate collapse to the definitions used by Jaber and Webb (1989).

Table 1 Comparison of cooling tower heights estimated by present method and rigorous method

T_{Gin}	Z_s		Z					
	T_{Lin1}	T_{Lin2}	$r = 1.0$		$r = 0.9$		$r = 0.8$	
			T_{Lin1}	T_{Lin2}	T_{Lin1}	T_{Lin2}	T_{Lin1}	T_{Lin2}
27	0.464	0.785	0.422	0.762	0.428	0.772	0.434	0.782
29	0.498	0.849	0.463	0.832	0.465	0.845	0.468	0.838
31	0.557	0.926	0.513	0.918	0.512	0.911	0.507	0.904
33	0.619	1.021	0.575	1.026	0.564	1.004	0.554	0.984
35	0.707	1.143	0.655	1.168	0.632	1.123	0.610	1.081

T_{Lin1} = 35°C; T_{Lin2} = 40°C; L = 1.0 kg/(m²s); G = 1.0 kg/(m²s); h_G = 47.09 W/(m²C); h_L = 3975 W/(m²C); a = 42 m²/m³; K_y = 0.054 kg/(m²s).

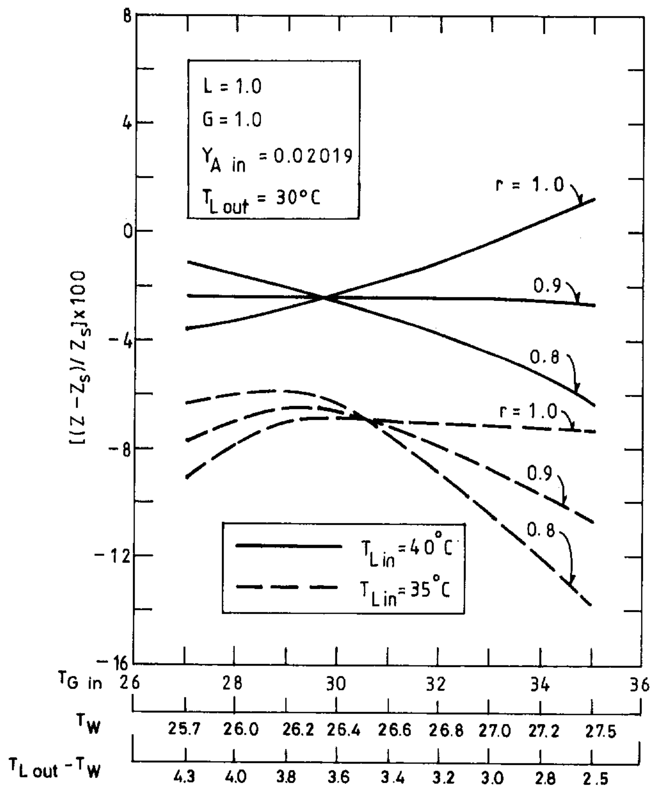

Fig. 2 Effect of inlet air temperature on required height

From the operating diagram of the cooling tower, Fig. 1, it is obvious that the maximum heat that can be transferred without violating the Second Law of Thermodynamics is

$$q_{max} = m_{min}(H_{Giout}^* - H_{Gin}^*) \tag{6}$$

The effectiveness ϵ can now be defined as $\epsilon = q_{actual}/q_{max}$ and related to the NTU (number of transfer units) by (as described by Jaber and Webb, 1989)

$$\epsilon = \frac{1 - \exp[-\text{NTU}(1 - C_R)]}{1 - C_R \exp[-\text{NTU}(1 - C_R)]} \tag{7}$$

where NTU $= k_y \cdot a_m / m_{min}$; C_R (capacity rate ratio) $= m_{min}/m_{max}$; m_{min} and m_{max}: minimum and maximum of $L \cdot C_{pL}/f'$ and $\{G_B(b_0 + \beta)\}/\{(\alpha + 1)B''\}$.

Results and Discussion

Numerical results were obtained for different inlet conditions for a countercurrent flow cooling tower using the modified ϵ–NTU method described above. The height of the column required to accomplish a prescribed cooling duty was determined at different liquid to air ratios (L/G), at various values of r. The height (Z) obtained from the modified ϵ–NTU method is compared with the height (Z_s) obtained from a rigorous method described by Treybal (1984), which is used as a benchmark. This is shown in Table 1 and also in Fig. 2. The rigorous method requires the tower to be divided into a very large number of segments over which the mass and energy balance equations coupled with the rate equations have to be solved over each differential segment sequentially. This procedure obviously requires the use of a computer. The heat and mass transfer coefficients of the 1 in. Berl saddles packing (fill) required for the computations were obtained using the procedures described by Treybal (1984). The ϵ–NTU method requires the assumption that the saturated enthalpy (or apparent enthalpy) versus temperature relation is linear, which is not strictly true. Jaber and Webb (1989) have shown that if the tower is divided into up to three segments and the ϵ–NTU method applied over each segment, the linearity assumption is not violated.

In Fig. 2, the effect of air inlet temperature on the height obtained by the present method when compared with the rigorous method is shown. It is seen from the figure that up to a particular air inlet temperature, the underprediction of the column height decreases at all r values at the lower cooling range and at the lower values of r at the higher cooling range. However, in both cases, the curves for different values of r meet at a particular point when the trend is reversed. This indicates that at this point r does not have any significant effect. This can be explained by examining the definition of r, which has the gas side heat transfer coefficient in the numerator and the mass transfer coefficient in the denominator. These coefficients are obtained from the hydrodynamic conditions in the tower and the type of tower internals. At the point where the curves meet, even though h_G has a finite value, the temperature driving force approaches zero and therefore all the cooling takes place only due to mass transfer. Beyond this point the trend is reversed as the contribution of the sensible heat transfer to the net cooling may be in the reverse direction. This can be observed from the rigorous method.

From a thermodynamic point of view it is possible to cool water of a temperature less than the entering air dry bulb temperature. This is because the actual or apparent enthalpy is for most practical purposes a function of adiabatic saturation temperature, or for air–water systems, the wet bulb temperature. It is also possible to operate a tower with entering air saturated, so long as its temperature is less than the water outlet temperature when the driving force is the wet bulb temperature approach. In Fig. 2 as the air inlet temperature increases, keeping its humidity constant, the wet bulb temperature of the air increases. Since the air inlet temperature is less than the water outlet temperature, cooling takes place.

Computations indicate that the L/G ratio does not have any effect on the change in heights obtained from the rigorous method and the ϵ–NTU method at all values of r. This is in accordance with the results of Sutherland (1983) and Jaber and Webb (1989).

Summary

The driving potential available for net heat transfer in countercurrent cooling towers has been expressed in terms of an apparent enthalpy. This apparent enthalpy is not limited by the Lewis relation, i.e., r need not necessarily be one. While this apparent enthalpy driving potential can be used in graphic methods as well as analytical methods of solution for cooling tower design, the present study has demonstrated its application in the ϵ–NTU method of design.

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Unsteady Buoyancy Exchange Flow Through a Horizontal Partition

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Nomenclature

$A = L/D$ = vent aspect ratio
D = width of the vent
g = acceleration due to gravity
H_1 = width of the enclosure
H_2 = height of the enclosure
L = height of the vent
Pr = Prandtl number
Q_{nd} = nondimensional total heat transfer across the vent
Q^* = normalized form of Q_{nd}
$\mathrm{Ra} = g\beta\Delta T_i D^3/\alpha\nu$ = Rayleigh number
Ra_{cr} = critical value of Rayleigh number for onset of flow through the vent
T_{cold} = initial temperature of fluid in upper chamber
T_{hot} = initial temperature of fluid in the lower chamber
T_{ref} = reference temperature
$\Delta T_i = T_{\mathrm{hot}} - T_{\mathrm{cold}}$ = initial reference temperature difference between upper and lower chamber
$U = uL/\alpha$ = dimensionless component of velocity in x direction
$V = vL/\alpha$ = dimensionless component of velocity in y direction
X, Y = dimensionless horizontal and vertical coordinates
α = thermal diffusivity of the fluid
β = coefficient of thermal expansion
ν = kinematic viscosity of the fluid
$\theta = \dfrac{T - T_{\mathrm{ref}}}{\Delta T_i}$ = dimensionless temperature
$\tau = t\alpha/D^2$ = dimensionless time

Introduction

Flow through apertures connecting two enclosures has been a subject of study for the last thirty years. Early studies were limited to openings in vertical partitions. The fundamental difference between flow through a vent in vertical partition and the flow through a horizontal partition is the stable stratification of the fluid in the former case while the configuration is unstable for the latter. As a result, in flows through a vertical partition, the lighter fluid layers move over heavy fluid layers with minimal interaction.

Brown (1962) was the first to study the flow through square openings in a horizontal partition with the heavier fluid above

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