Packed bed:

Consider a catalyzed reaction occurring in a packed bed. Mass transfer is the rate limiting step and the reaction follows first (or second...) order kinetics. Determine the conversion as a function of distance. Catalyst surface area per unit volume of reactor is given by \( a_c \). In the packed bed, the mass transfer coefficient can be obtained using \( Sh' = \left( \frac{Re}{1} \right)^{\frac{1}{2}} (Sc)^{\frac{1}{3}} \), where \( Sh' = \frac{k_m d_p}{D_A} \frac{\epsilon}{(1 - \epsilon)^{\gamma}} \),

\[
Re' = \frac{d_p V \rho_A}{\mu \left(1 - \epsilon \right)^{\gamma}} \quad Sc = \frac{\mu}{\rho_A D_A}
\]

Here, void fraction \((\epsilon)\) and shape factor \((\gamma = 1 \text{ for sphere})\) also play a role.

Solution:

\[
\frac{dF_A}{dV} = r_i a_c = -k_m C_A a_c
\]

\[
QC_{\text{lin}} \frac{dx}{\pi d^2/4} \frac{dz}{dz} = k_m C_{\text{lin}} (1 - x) a_c
\]

\[
\frac{dx}{(1 - x)} = \frac{k_m a_c}{V_s} dz
\]

\[
\ln(1 - x) = -\frac{k_m a_c}{V_s} L
\]

Example: (problem 11.5B in Fogler). In a packed bed, chlorine is removed by passing over spherical absorbent particles. Mass transfer is the rate limiting step, since upon contact with the surface, chlorine reacts immediately. At present, the removal rate is 63.2 %. If the flow rate is increased 4 times and particle size is decreased 3 times, and packed bed length is increased by 50%, what will be the removal?

\[
\ln(1 - x_1) = -\frac{k_m a_c}{V_{s_1}} L_1 = \ln(1 - 0.632) = -0.99967 = -1
\]

If the flow rate is increased 4 times, \( V_{s-2} = 4V_{s-1} \)

Bed length is increased by 50 %, \( L_2 = 1.5L_1 \)
And particle size is decreased 3 times

\[
\left( \frac{k_{m2} d_{p2}}{D_A} \right) \frac{\varepsilon}{(1 - \varepsilon)\gamma} = \left( \frac{d_{p2} V_{s2} \rho_A}{\mu} \right)^{\frac{1}{2}} \left( \frac{1}{(1 - \varepsilon)\gamma} \right)^{\frac{1}{2}} \left( \frac{\mu}{\rho_A D_A} \right)^{\frac{1}{3}}
\]

\[
\left( \frac{k_{m1} d_{p1}}{D_A} \right) \frac{\varepsilon}{(1 - \varepsilon)\gamma} = \left( \frac{d_{p1} V_{s1} \rho_A}{\mu} \right)^{\frac{1}{2}} \left( \frac{1}{(1 - \varepsilon)\gamma} \right)^{\frac{1}{2}} \left( \frac{\mu}{\rho_A D_A} \right)^{\frac{1}{3}}
\]

\[
\left( \frac{k_{m2} d_{p2}}{k_{m1} d_{p1}} \right) = \left( \frac{d_{p2} V_{s2}}{d_{p1} V_{s1}} \right)^{\frac{1}{2}}
\]

\[
\left( \frac{d_{p2}}{d_{p1}} \right) k_{m2} = k_{m1} \left( \frac{V_{s2}}{V_{s1}} \right)^{\frac{1}{2}} \left( \frac{d_{p1}}{d_{p2}} \right)^{\frac{1}{2}} = k_{m1} 2\sqrt{3} = 3.46k_{m1}
\]

\[
\ln(1 - x_2) = \frac{-k_{m2} a_{c2}}{V_{s2}} L_2 = \frac{-k_{m1} a_{c1}}{V_{s1}} L_1 \times \frac{3.46 \times 3 \times 1.5}{4} = -1 \times 3.8925 = -3.89
\]

\[
x_2 = 0.98
\]