1. Consider the unsteady flow of an incompressible Newtonian fluid between two cylinders, caused by sudden movement of outer cylinder with a constant linear velocity of $V_0$ as shown in the figure. (None of the cylinders are rotating. Outer cylinder is moving in the z direction with velocity of $V_0$. You can assume that the cylinders are of infinite length and hence end effects can be neglected. No slip conditions apply).

![Diagram of unsteady flow between two cylinders](image)

Determine the velocity profile as a function of radius and time


A part of a lubrication system consists of two circular disks between which a lubricant flows radially. The flow takes place because of a modified pressure difference $P_1 - P_2$ between the inner and outer radii $r_1$ and $r_2$ respectively.

![Diagram of radial flow between disks](image)

(a) Write the equations of continuity and motion for this flow system, assuming steady-state laminar incompressible Newtonian flow. Consider only the region $r_1 \leq r \leq r_2$ and a flow that is radially directed.

(b) show how the equation of continuity enables one to simplify the equation of motion to give

$$
\frac{\rho \phi^2}{r^3} = -\frac{dP}{dr} + \mu \frac{1}{r^2} \frac{d^2 \phi}{dz^2}
$$

in which $\phi = r V_r$ is a function of $z$ only. Why is $\phi$ independent of $r$?
(c) It can be shown that so solution exists for the above equation unless the nonlinear term containing \( \phi \) is omitted. Omission of this term corresponds to the “creeping flow assumption.” Show that for creeping flow, the above equation can be integrated with respect to \( r \) to give
\[
0 = P_1 - P_2 + \left( \mu \ln \frac{r_1}{r_2} \right) \frac{d^2 \phi}{dz^2}
\]

(d) Show that further integration with respect to \( z \) gives
\[
V_z(r, z) = \frac{(P_1 - P_2)b^2}{2 \mu \ln \left( \frac{r_2}{r_1} \right)} \left[ 1 - \left( \frac{z}{b} \right)^2 \right]
\]

(e) Show that the mass flow rate is
\[
w = \frac{4\pi (P_1 - P_2) b^3 \rho}{3\mu \ln \left( \frac{r_1}{r_2} \right)}
\]